Analysis of Crack Propagation Stability under a Diaxial Stress Field

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1. Abstract

In this manuscript an analysis of the crack propagation process is performed. For this purpose, experimental results on SEN specimen made of PMMA are considered. Also, the value of the second coefficient of Williams stress series is taken as a fracture criterion. The whole fracture process is explained with this parameter in conjunction with the maximum circumferential stress and the energetic criteria. The results are explained with the analysis of the stress field at the crack tip.

2. Resumen

En este trabajo de investigación se analiza el proceso de propagación de grietas. Para esto, en el análisis experimental se utilizó un espéscimen tipo SEN de un material PMMA. Además, se consideró el segundo coeficiente de la serie de Williams como criterio de fractura. Los resultados fueron comparados con el criterio del Esfuerzo Circunferencial Máximo y el criterio Energetico.

3. Introduction

One of the main problems in Fracture Mechanics, is the analysis of the fracture process under arbitrary loads and the evaluation of crack propagation stability. It is important to remember that there are two conditions that influence the behavior of the crack propagation, namely local conditions and global conditions; the first involve the flaws and the imperfections of the material which are in the crack path, while in the second, the specimen geometry, the applied loads and the environment are some of the parameters that influence the fracture process. Likewise, if the fracture process is dominated by the local conditions, one can say that the process is uncertain and, if the global conditions prevail, the crack growth is predictable. This second case is of interest. Williams [1] developed a function that describes the stress field in the vicinity of the crack tip (figure 1).

\[ X_r(r, \psi) = \sum_{n=1,2,3,\ldots} \left[ (-1)^{n-1} A_n r^{-n+1/2} \left\{ -\cos\left(n - \frac{3}{2}\right) \psi + \frac{2n - 3}{2n + 1} \cos\left(n + \frac{1}{2}\right) \psi \right\} + \left[ (-1)^{n-1} A_n r^{n+1/2} \left\{ \cos\left(n - \frac{3}{2}\right) \psi + \cos\left(n + \frac{1}{2}\right) \psi \right\} \right] + \sum_{n=1}^{\infty} \left[ (-1)^{n-1} B_n r^{n+1/2} \left\{ \cos\left(n - \frac{3}{2}\right) \psi - \cos\left(n + \frac{1}{2}\right) \psi \right\} \right] + \left[ (-1)^{n-1} B_n r^{n+1/2} \left\{ \cos\left(n - \frac{3}{2}\right) \psi + \frac{2n - 3}{2n + 1} \cos\left(n + \frac{1}{2}\right) \psi \right\} \right] \}

The boundary conditions along the crack faces are satisfied and the constants \( A \) and \( B \) are unknown.

Later, Cotterell [2] analyzed the symmetrical terms of the stress field considering only mode I. Also, it was showed that the first term of the series of Williams determines the Stress Intensity Factor for a fragile material, the second controls the crack directional stability, the third is governed by the propagation stability and the four determines if the maximum stress on the crack propagation increases or diminishes.

Due to this, Cotterell [2], did consider a distance \( r \) from the...
crack tip, and through mathematical analysis obtained an equation which related the original angle of crack propagation $d\theta$ and the far away extension angle $d\phi$ (figure 2). Moreover, he established that if $A_x$ is negative $d\theta > d\phi$ crack propagation is stabilized along the original path, and if $A_x$, it positive $d\theta < d\phi$ the crack spreads away from the original route.

\[
d\theta = \left[ 1 - \frac{4}{\pi} \frac{A_2}{A_1} \sqrt{\frac{s}{r}} \right] d\phi
\]

This criterion was also applied by other investigators as Benbow and Roessler [3], Guenney and Guilmain [4], Larsson and Carlsson [5], who demonstrated this relationship experimentally.

Also, crack propagation angles were evaluated experimentally in a wide range of SEN specimens under biaxial loading. This behavior was compared with the second coefficient of Williams, which was evaluated with the code ANGCRRK [6]. The above statements were also observed. The main limitation of this analysis is that depending on the $A_x$ value, it can be stated if there is stability in the crack process. However, it is not possible to evaluate the crack propagation angle.

This can be solved numerically with the FEM in the following way. After an initial evaluation of $K_x$ and $K_y$, the energetic criteria and the maximum circumferential stress criteria were applied. With each of these criteria, the crack propagation angle is evaluated.

At this point, two procedures were followed. In first instance, ANSYS code [7] was used. In this case, after the crack propagation angle was evaluated, the crack was propagated by small steps in the direction evaluated. In the second case, the FRANC2D/L code [8] was used. In this case the crack propagation is modelled by the computer.

4. Development

4.1 Numerical Analysis

A numerical analysis was made with the following experimental results reported in [9]. For this purpose, two SEN specimens at 152mm x 102mm x 6mm of PMMA were studied, with the loading and geometrical conditions shown in table 1. The numerical results are summarised in table 2 and, at the same, they are compared with experimental results. As it can be seen, there is a close agreement. At this point, it is important to mention that, in the ANSYS evaluation the crack is propagated by steps, thus, there is the possibility of accumulated errors in the analysis. However, this situation did not play an important role.

In the case of specimen $A$, the second coefficient of Williams at crack initiation is 7.58 and the value of this parameter at the end of the crack process observed is 9.6. Figure 4 shows a detail of the stress field at this last stage. As it can be seen, there is an asymmetry which clearly shows that the instability will continue, this explains why the value of the second coefficient has growth.

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crack initiation is 7.58 and the value of this parameter at the end of the crack process observed is 9.6. Figure 4 shows a detail of the stress field at this last stage. As it can be seen, there is an asymmetry which clearly shows that the instability will continue, explaining why the value of the second coefficient has growth. In the case of specimen B, the value of the second coefficient of Williams at crack initiation is 2.58. This is lower than those calculated in specimen A. The analysis of the stress field still shows a certain degree of asymmetry (figure 5).

As the crack propagates the symmetry is restored. This indicates that the propagation is nearly stabilized. When the crack length is 56 mm, the value of the second coefficient is 1.6. This is confirmed with the details of the stress field being virtually symmetric at the crack tip (figure 6).

Summarizing, the analysis of the variation of the second coefficient of the series of Williams clearly shows that as this parameter decreases, the symmetry of the stress field is restored and crack propagation stability is developed. Moreover, the conditions obtained with the evaluation of the second parameter of Williams are complemented with a FEM of the stress field at the crack tip in conjunction with the maximum circumferential stress of the energetic criteria.

Another aspect, that was observed is that a bigger compressive transverse load stabilises the crack propagation path. This is confirmed when the crack angle is compared with the compressive loads, which were applied to both specimens. In other words, in both specimens, their dimensions and transverse loading conditions were nearly maintained constant. Nevertheless, the transverse loading was increased in specimen B. As a result, the crack propagation angle was reduced in the last case. This confirms the important role that is played in the fracture process by these compressive loads.

5. Conclusion

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6. References


