The core vote effect on the annulled vote: an agent-based model

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Abstract— Using an agent-model, we conducted an analysis of a two-party system, in order to identify a pattern of behavior in the influence exercised by the number core votes on the proportion of the number of annulled votes, which are cast during an election. In a 2d toroidal grid network, where each node is connected with its four closest neighbors, we introduce the concept of an Opinion Network with Uniformly Distributed Nodes that can have one of three different states, also known as “spin”: +1, -1 and 0. Initially, the states +1 and -1 correspond respectively to “active votes” for candidate A and candidate B, with the specific feature that these are not able to change their state “core vote”; that is, who do not change their political preferences. The rest is established as having a state of 0, known as “undecided voters”, who will try to become “active votes” in favor of one of the two contending candidates, during the process of interaction with their four closest neighbors (because of their influence, in real life these would be interpreted as the voters’ immediate surroundings). The model dynamics tend to a state of equilibrium in which there are no more voters changing their opinion upon interaction with his four closest neighbors. Finally, active voters who decide become part of the valid vote, while the undecided become part of the annulled vote.

Keywords— Opinion networks, majority rule, adaptive behavior, core vote, valid vote.

INTRODUCTION

Elections are one of the fundamental processes in democratic societies. From a more scientific point of view, the result of an electoral process can be considered a response from an open system that contains diverse elements that interact and that is conditioned by complex dynamics. Voters do not have much freedom, even in democratic regimes, because individual preferences depend greatly on the choice of social networks, in which the voter participates. This is a normal human behavior in response to the different social identities that can lead to social tensions; therefore, the microsociocultural element is a good place to reduce tensions through adjusting political preferences or ideological currents. According to this reasoning, a quantitative characterization of an electoral network can be performed using a vote-distribution study. These vote distributions are obtained from various electoral processes and subsequent simulations of electoral preferences, which are indispensable steps toward a better understanding and prediction of underlying the electoral dynamics.

Voting is a manifestation of one’s own will or opinion about a subject, a person or a group of people and is regulated by a certain entity; this manifestation is delivered under a normalized form, without an explicit justification and leading towards a collective decision. Voting implies the existence of a group of people eligible to vote and a previously specified issue, on which the desire of the group of people will be expressed. Voting can be secret or public, direct or indirect; or simple (one voter, one vote), plural (one voter, various votes); or cumulative (as many votes as candidates to choose from). When there is a vote for one of the options being put forward, we call this an active vote.
According to Sirvent and Delgadoillo (1997), various empirical studies on the behavior of the electorate in modern democracies have demonstrated that there are levels in each population with considerable stability in their preferences, which indicates that an individual vote has high predictability in its political behavior over time. Thus, we may call this, ‘stable party identification’, also known as the “core vote,” this is to say that each political party has a stable electoral base derived from the structural conditions of the party. At the other extreme, we find abstention and the annulled votes. Both of these phenomena are focused on the expression by the voter of disagreement with or indifference to the ideas of the contending candidates. On the one hand, when abstaining, the voter decides not to exercise his or her right to vote, which is in contrast to the annulled votes, when the voter decides to exercise his or her vote by annulling any preference for competing candidates.

In democratic political systems, there are campaigns for vote nullification, in which one of the consistent features of these actions is the voters’ claims of non-conformity with the ideas of the proposed candidates (Ackerman John 2011; Driscoll et. al 2013). Usually, these ideas are proposed by people involved in politics, who are manifesting their opinion through voting; i.e., active voters. Finally, the voter conducts a premeditated political critique, and many opt for vote nullification intentionally. This type of vote is justified because it ensures citizen’s freedom of expression through the right to vote. In countries where voting is not mandatory, intentional annulled vote is considered the voter’s civic act in order to express his or her disagreement with the ideas of eligible candidates. The treatment of voting, however, is regarded differently depending on the country and its legal system and the Law gives voting a special treatment (Timothy J. et al., 2007). For example, in Spain, votes are counted, but they are not considered to designate Party’s representatives; in France, votes are not counted; in Colombia, votes are considered for the designation of elected office public posts by means of proportional representation; in Ecuador, voting may give way to an annulment of the election, etc. (Vázquez José 2012).

I. Development

To model the SOD with three states, we use a simple variation of MR in a homogenous 2d toroidal grid network with \( n^2 = N \) nodes, such that each node \( \{ 1 \ldots N \} \) only connects to its corresponding four closest neighbors and is initially distributed among the three states \( i = +1, -1, 0 \); nodes with states \( +1 \) and \( -1 \) are distributed randomly on the whole network, while nodes with state \( 0 \) are distributed in the rest of the population; in the context of social dynamics, we define \( +1 \) to be preference for candidate A and \( -1 \), preference for candidate B, with the condition that voters initially preferring a certain candidate cannot change their opinion; while nodes with the \( 0 \) state are defined as active undecided voters or simply undecided voters. This paper considered as a cycle the process conducted for \( N \) interactions between voters.
Changing only the state of the selected node $a_{i,j}$, the majority rule changes (see Fig. 4). It is important to highlight this difference, because the rule we use prevents the formation of fringes (Chen P. and Redner S., (2005); Balankin et al. 2011), which consequently prevents slow kinetics. In this last stage, the active voters with a preference for a given candidate and core voters become part of the valid voting; while the undecided voters become part of the annulled voting (see Fig. 5).

We report the results of an average of 100 simulations for each case and networks of size $400 \leq N \leq 10,000$ used.

**BEHAVIOR OF VOTERS WITH A PREFERENCE**

We focus on the concentration of core voters with a preference for candidate B ($V_{bi}$) and the concentration of final voters for candidate B ($V_{bf}$). We conducted a variation of undecided voters ($V_{0i}$), in function of the percentage of initial undecided voters for candidate B ($V_{bi}$), maintaining the percentage of core voters with a preference for candidate A ($V_{ai}$) at just one point; i.e.:

$$V_{0i} = 100\% - V_{ai} - V_{bi} \quad (1)$$

with fixed ($V_{ai}$)

As a specific case, see point $V_{ai} = 50\%$ that varies $V_{bi}$ in an interval of $[0, 0.5]$. Under these initial conditions and applying the model dynamics to conduct Monte Carlo simulation, we obtained the final preference for candidate B, described by the following equation:

$$V_{bf} = f(V_{ai}) = \frac{\alpha}{\sum_{i=0}^{\infty}}$$

(2)

The standard deviation of 100 simulations for each point for reported results is 0.5%, which gives an error standard of 0.05%.

Figure 6 graphically describes the behavior for different size networks for a 9-neighborhood. We observe that the behavior holds regardless of the size of the network. Evidently, when $V_{bi} = 50\%$, then $V_{bf} = V_{bf} = 50\%$. In the case of a 9-neighborhood, we found that the behavior is preserved, because the dynamics only affects the selected node in the corresponding iteration, no matter how many neighbors are
considered. However, it would become necessary to make a more detailed analysis of a future generalization of this model, because in the current one, the preservation of the behavior cannot be guaranteed for $n$-neighbors, with $2 \leq n \leq N$.

Figure 6. Final percentage of voters compared with their initial state in networks of different size with $V_{ai}$ fixed at 0.5N.

More generally, we observed that when the fixed concentration of $V_{ai}$ changes, there is different growth in the percentage of final voters for candidate B ($V_{bf}$) maintaining the relation given by changes in parameters $a$, $b$, $c$ described in Table 1 and Figure 7, with Equation 2.

Table 1. Adjustment parameters for Equation 1, depending on the initial percentage of core votes $V_{ai}$.

<table>
<thead>
<tr>
<th>$V_{ai}$</th>
<th>2.78%</th>
<th>8.33%</th>
<th>16.66%</th>
<th>25%</th>
<th>33.33%</th>
<th>41.66%</th>
<th>50%</th>
<th>58.33%</th>
<th>66.66%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>115.86</td>
<td>36.34</td>
<td>14.33</td>
<td>7.24</td>
<td>4.32</td>
<td>2.84</td>
<td>2.06</td>
<td>1.38</td>
<td>1.29</td>
<td>1.13</td>
</tr>
<tr>
<td>$b$</td>
<td>117.37</td>
<td>38.21</td>
<td>15.84</td>
<td>8.15</td>
<td>4.76</td>
<td>2.87</td>
<td>1.82</td>
<td>1.10</td>
<td>0.63</td>
<td>0.35</td>
</tr>
<tr>
<td>$c$</td>
<td>1.36</td>
<td>1.44</td>
<td>1.47</td>
<td>1.43</td>
<td>1.38</td>
<td>1.28</td>
<td>1.20</td>
<td>1.12</td>
<td>1.07</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Figure 7. Adjusting the parameters to characterize the behavior of $V_{bf}$

As a particular case, we distributed the same quantity of core voters for A and B, using a random process with a uniform distribution; and in this case, both groups have the same likelihood of ending up with a larger number of voters with a defined preference. However, the most interesting features can be found in the case of undecided voters, as it is shown below.

BEHAVIOR OF UNDECIDED VOTERS

In theory, annulled votes are in terms of final active voting; i.e.: $V_{0f} = 1 - V_{af} - V_{bf}$ (3)

Figure 8 shows the percentage of final annulled votes in function of the distribution of initial core voters with preference. Figure 5 and Equation 3 show that the behavior is independent from network size. But as Figure 7 shows, this behavior does depend on the number of voters, initially distributed, with a defined preference. Through a numeric simulation and replacing 2 on 3, we found that:

$$V_{0f} = 1 - \frac{a}{V_{ai}} - \frac{b}{V_{ai}} - \frac{c}{V_{ai}}$$ (4)

To perform the analysis, we establish a quantity ranging from 10% to 90% of voters for candidate A, and for each fixed quantity of voters for candidate A, we vary the initial quantity of voters for candidate B from 90% to 10%. We can then observe that the number of annulled votes changes depending on the initial distribution of core voters (Figure 9).
Once the numeric simulation has been analyzed, we observe that no more than 5.97±0.08% of the final population chose to annul their vote. This happens when we initially distribute 33% of voters for A and 33% of voters for B, leaving 34% of voters not committed (see Figure 9), where 0.08% is the standard error of 100 samples, with a 95% trust level.

Since its behavior is described in function of $V_{af}$ and $V_{bf}$, it does not depend either on network size (see Figure 10).

We showed the general mapping with the intention of getting to know all the possible scenarios.

Aside from network size, all curves show the same slope (Figure 11), which leads us to find growth $T_e$ regarding network size. The number of cycles, however, varies with respect to the same network, depending on the number of initial voters. Then, if we make the necessary adjustments, we have:

$$ T_e = f(N, V_i) = -41.72 + 42.71V_i^{0.06} + 2.88V_i^{1.13} $$

Where $V_i^{0.06}$ is the part that depends on the network size $N$, and $2.88V_i^{1.13}$ is the part that depends on the initial core votes $V_i$ independent of the candidate.

**II. CONCLUSIONS**

In the dynamics of social systems, there are different real variables on which the modeling depends; however, the model developed in this study was considered by his simplicity and because the main purpose of this paper is showing an initial approach for explaining the behavior of intentional vote annulment in elections, using the core vote based on a democratic two-party electoral environment, without treating it as realistic at all. Broadly speaking, we only consider in a fictitious way the interpretation of a model that includes effective voting in elections, because the people who actively
vote are usually the people who interact with their environment to make decisions. We suggest that annulled voting depends in part on the opinion surroundings in which the voters are immersed.

The methodology used here is not based on strictly theoretical approaches; instead, it is based on heuristic approaches in a similar manner that was done in P. Chen and S. Redner, (2005) or Balankin, et. al. (2011). Also, the probabilistic behavior of the system was described by using Montecarlo simulations and the inclusion of equations, which seem not to show any obvious information, is justified because they could be used as a reference for any reader interested in developing a strictly theoretical result for this topic; so, in our development we did not do it in that way, because we have always been aware of the difficulties and the complexity that it would have implied for a primal result.

In order to obtain more realistic results for future research, it is considered to try to complement the dynamics of this model with the following considerations: adding effects on the spatial distribution of the voters, generalizing the model for n different opinions, implementing coalitions between the different opinions and analyzing the different rules in the dynamics to favor the lowest opinion, the highest opinion or simply favoring none of them; or generalizing the model in d dimensions. Using the reference [Balankin, et. al. 2011], where it is shown the big effect of different distributions of initial voters. Or authors, such as Timothy J Power and James C. Garand (2007), have explored the effects on the annulment of votes in terms of institutional, socioeconomic and protest-democracy variables.

Finally, we remark that the main objective of this paper is just looking for a primal and simpler explanation of how two different and apparently unrelated phenomena, such as the core voting and the annulled voting, are related to each other. Corresponding Monte Carlo simulations were modeled and conducted for the simplest cases, where voters were distributed randomly on the whole network. Participants with a core voting affect the final concentration of voters with their potential preference (Eq. 2), which coincide with self-organizing systems of complex systems where agents interact in an uncertainty environment. Their effect on the active voter without a preference is explained by the interactions of the nodes with their neighbors over a determined number of times (cycles); however, because they are equally surrounded by participants with different preferences, voters opt for annulling their vote, with a higher concentration of annulled votes, when the initial concentration of core voters with a preference amounts to 33% for both parties, leading to a maximum level of 5.97±0.08% of undecided voters. In contrast, the stability is affected by network size; however, the maximum number of interactions that a participant has to undertake to make a decision is on the order of 10^4, similarly to the model exposed by [Galam S. 2002]; but in this case its growth is proportional to the network size and the initial percentage of core voters, independent of the candidate with a behavior based on Equation 5 (Figure 11).

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IV. REFERENCES


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